Influence of Inclined Holes in Measurement of Residual Stress by the Hole Drilling Method

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The hole drilling method is widely used in measuring residual stress in surfaces. In this method, the inclination of holes is one of the sources of error. This paper presents a finite element analysis of the influence of inclined holes on the uniaxial residual stress field. The error in stress has been found to increase proportionally to the correct inclined angle of the hole. The correction equations by which one may easily obtain the residual stress, taking account of the inclined angle and direction, have been derived. The error of stress due to the inclined hole has been reduced to around 1% using the correction equations.

Key Words : Hole Drilling Method, Finite Element Analysis, Residual Stress, Inclined Hole

1. Introduction

Residual stress means the stress that exists inside machinery or structures with no external loads. Such residual stress is superposed onto the service stress and affects the fatigue life of machinery or structures considerably. Especially, for structures that are connected with a material part that had gone through hot forming procedure or welding, residual stress may increase to the level of yield stress. For such a reason, if the structures were designed on the assumption of no residual stress, significant safety problem may exist. As a result, the magnitude of residual stress should be accurately considered in the design of structures, and accurate measurement of residual stress is very important.

The hole drilling method makes a little hole in the metal surface that has residual stress and measures the relieved stress with a strain gage. It is used widely in measuring the residual stress in surfaces. Mathar (1934) first suggested this method in the 1930s, and constant developments were achieved in its theory and applications. Further studies on more accurate and simpler measurement instruments and methods are being accomplished (Flaman et al., 1986 ; Tootoonian et al., 1995 ; Schajer et al., 1997).

The current standard examination method of the hole drilling method is specified in ASTM E837-92. Because the measurement figures vary by the locations of the holes, vertical hole drilling is assumed in ASTM, and the eccentricity between the center of the strain gage and the center of the hole is restricted to be under 0.025mm. However, in many cases of machinery and structures, measurement objects are often irregular, and even when specific measurement tools such as RS-200 are used, accurate vertical hole

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drilling is quite difficult to perform. As a result, when holes are drilled with a gradient, errors are generated by the inclined hole. Therefore, when such hole drilling is carried out, it is required that some error should be anticipated in advance, and a method that can compensate for the error is required. Besides, as it is impossible to remeasure at the identical location with a hole drilling method, it is required to utilize a method that can correct the measured figures, even if the hole was drilled in a little slanted position.

Studies on the hole drilling method until now were conducted mainly on the technique of hole drilling (Flaman et al., 1986 ; Tootoonian et al, 1995), on how to decide the calibration coefficients (Schajer, 1981, 1988 ; Vangi, 1994 ; Joo et al., 1998), and on the error of measurement (Ajovalastt, 1979 ; Schajer et al., 1996 ; Kim et al., 2000). Besides, there were a number of studies on measurement of residual stress using the hole drilling method (Yen and Lin, 1995 ; Yang et al., 1998 ; Seok et al., 1999). Among these, studies on the error of measurements were carried out mainly on the effect of the hole eccentricity, but studies on the inclined hole have scarcely been done. On the other hand, by applying an finite element analysis to the hole drilling method, we can set the known residual stress field accurately, making the error analysis more effective.

As a result, in this study, a uniaxial residual stress field is modeled in the finite element method, and the size and tendency of errors due to inclined holes are found. Also, we propose equations that can be used to correct the residual stress measurement, and then check the efficiency of the proposed equations.

2. Calibration Coefficients

The calibration coefficients are a factor that is related to the measured strain and residual stress when we drill holes in the residual stress field. They vary according to the depth and size of the hole and the material property. Many researchers have suggested methods for the determination of the calibration coefficients.

2.1 Calibration coefficients for through holes

The calibration coefficients when drilling through holes are calculated from an equation that is drawn from the equation based on the solution of Kirsch (1970). If a uniaxial tensile stress σ_x exists on the plate, in Fig. 1, we can express the stress field at an arbitrary point $P(R, \alpha)$ by using a polar coordinate system in Eq. (1). When there is a through hole with a radius of R_0 in the center, as in Fig. 2, the stress field is theoretically identical to Eq. (2).

$$\begin{aligned} \sigma_r' &= \frac{\sigma_x}{2}(1+\cos 2\alpha) \\ \sigma_\theta' &= \frac{\sigma_x}{2}(1-\cos 2\alpha) \\ \tau_{r\theta}' &= \frac{\sigma_x}{2}\sin 2\alpha \end{aligned} \tag{1}$$

$$\begin{aligned} \sigma_r'' &= \frac{\sigma_x}{2}\left(1-\frac{1}{r^2}\right)+\frac{\sigma_x}{2}\left(1+\frac{3}{r^4}-\frac{4}{r^2}\right)\cos 2\alpha \\ \sigma_\theta'' &= \frac{\sigma_x}{2}\left(1+\frac{1}{r^2}\right)-\frac{\sigma_x}{2}\left(1+\frac{3}{r^4}\right)\cos 2\alpha \\ \tau_{r\theta}'' &= -\frac{\sigma_x}{2}\left(1-\frac{3}{r^4}+\frac{2}{r^2}\right)\sin 2\alpha \\ &\text{where } r=R/R_0 (R \geq R_0) \end{aligned} \tag{2}$$

As a result, the relieved stresses with the hole drilling process can be drawn from the difference between Eq. (2), which indicates the stresses after drilling the hole, and Eq. (1) that expresses the initial stresses; and they can be expressed as in Eq. (3).

$$\begin{aligned} \Delta\sigma_r &= \sigma_r''-\sigma_r' \\ \Delta\sigma_\theta &= \sigma_\theta''-\sigma_\theta' \\ \Delta\tau_{r\theta} &= \tau_{r\theta}''-\tau_{r\theta}' \end{aligned} \tag{3}$$

If the plate is homogeneous, isotropic, and linearly elastic, the relieved strains are expressed as in Eq. (4), by applying the Hooke's law to Eq. (3) and simplifying.

$$\begin{aligned} \varepsilon_r &= \sigma_x(A+B\cos 2\alpha) \\ \varepsilon_\theta &= \sigma_x(-A+C\cos 2\alpha) \end{aligned} \tag{4}$$

$$\text{where } A=-\frac{1+\nu}{2E}\left(\frac{1}{r^2}\right)$$

$$B=-\frac{1+\nu}{2E}\left\{\left(\frac{4}{1+\nu}\right)\frac{1}{r^2}-\frac{3}{r^4}\right\}$$

$$C=-\frac{1+\nu}{2E}\left\{-\left(\frac{4\nu}{1+\nu}\right)\frac{1}{r^2}+\frac{3}{r^4}\right\}$$

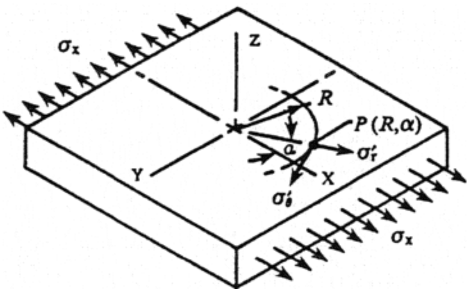


Fig. 1 Stress states at $P(R, \alpha)$ before the hole drilling

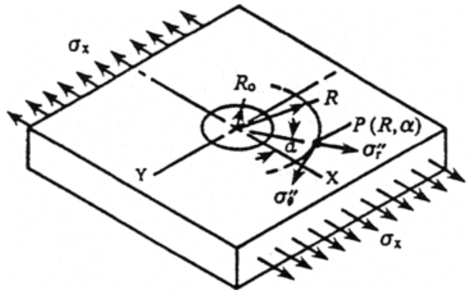


Fig. 2 Stress states at $P(R, \alpha)$ after the hole drilling

Generally, residual stress fields often take the form of biaxial stress distribution. As a result, when there is a uniform stress σ_x, along with uniform stress σ_y along the y axis, we can calculate the strain as in Eq. (5), by substituting $\alpha+90°$ for α in Eq. (4), as we did for the x axis.

$$\begin{aligned}\varepsilon_r &= \sigma_x(A+B\cos 2\alpha)+\sigma_r(A-B\cos 2\alpha)\\ &= A(\sigma_x+\sigma_y)+B(\sigma_x-\sigma_y)\cos 2\alpha \end{aligned} \tag{5}$$

By applying Eq. (5), a basic equation for measuring residual stress with the hole drilling method, at each location of the rosette strain gage in Fig. 3, we can find out the relieved strain at that location as in Eq. (6).

$$\begin{aligned}\varepsilon_1 &= A(\sigma_x+\sigma_y)+B(\sigma_x-\sigma_y)\cos 2\alpha\\ \varepsilon_2 &= A(\sigma_x+\sigma_y)+B(\sigma_x-\sigma_y)\cos 2(\alpha+45°)\\ \varepsilon_3 &= A(\sigma_x+\sigma_y)+B(\sigma_x-\sigma_y)\cos 2(\alpha+90°)\end{aligned} \tag{6}$$

In Eqs. (4) ~ (6), A and B are the calibration coefficients, and these indicate the relationship between the stress and strain that are relieved into the radius direction, when the hole is drilled.

2.2 Calibration coefficients for blind holes

In realistic situations, most of the machinery and structures have arbitrary size and thickness, so the measurement of residual stress is mainly carried out in drilling blind holes. The relieved strain calculation equation for a blind hole can be expressed by applying the calibration coefficients $\bar{A}$ and $\bar{B}$ of the blind hole to Eq. (6). The analysis of residual stress from blind holes takes one more variable into consideration, that is, z/D which is the ratio of the depth of the hole, z, to the gage radius D. It can be expressed in the form of general functions as in Eq. (7).

$$\begin{aligned}\bar{A} &= f_A(E,\ \nu,\ r,\ z/D)\\ \bar{B} &= f_B(E,\ \nu,\ r,\ z/D)\end{aligned} \tag{7}$$

There is no theoretical solution to the calibration coefficients of Eq. (7), $\bar{A}$ and $\bar{B}$, and they must be obtained through experimental methods or finite element analyses. However, because the calibration coefficients $\bar{A}$ and $\bar{B}$ are determined by the forms of strain gages once the measurement materials are decided, Schajer (1981) expressed $\bar{A}$ and $\bar{B}$ as in Eq. (8), by introducing new dimensionless coefficients a and b, which depend only weakly on Poisson's ratio and vary by less than 1% for the range of Poisson's ratio from 0.28 to 0.33.

$$\begin{aligned}\bar{A} &= -\frac{(1+\nu)}{2E}a\\ \bar{B} &= -\frac{1}{2E}b\end{aligned} \tag{8}$$

On the other hand, if the calibration stress σ_c is added to ε_r of Eq. (4), the relieved strain in the gage directions No. 1 ($\alpha=0°$) and No. 3 ($\alpha=90°$) can be expressed as in Eq. (9) by applying the calibration coefficients $\bar{A}$ and $\bar{B}$ of the blind hole. Eq. (10) shows how $\bar{A}$ and $\bar{B}$ are suggested.

$$\begin{aligned}\varepsilon_1 &= \sigma_c(\bar{A}+\bar{B})\\ \varepsilon_3 &= \sigma_c(\bar{A}-\bar{B})\end{aligned} \tag{9}$$

$$\begin{aligned}\bar{A} &= \frac{\varepsilon_1+\varepsilon_3}{2\sigma_c}\\ \bar{B} &= \frac{\varepsilon_1-\varepsilon_3}{2\sigma_c}\end{aligned} \tag{10}$$

As a result, through experimental methods or finite element analyses, we can calculate the calibration coefficients, $\bar{A}$ and $\bar{B}$, by drawing out the

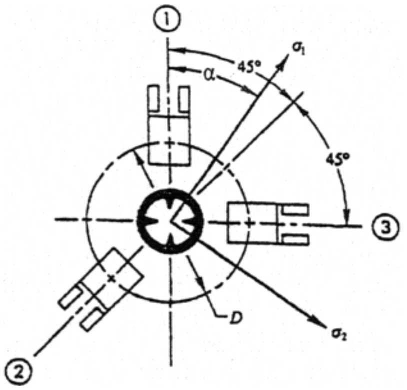


Fig. 3 Typical rosette strain gage for the hole drilling method

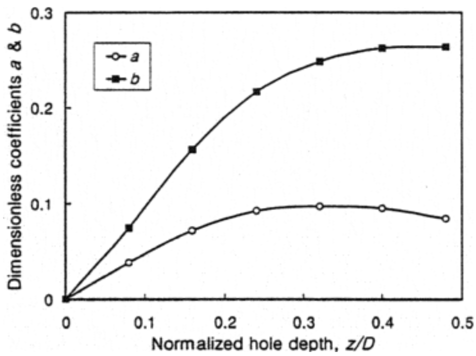


Fig. 4 Dimensionless coefficients, a and b, as functions of hole depth

unknown figures of Eq. (10) and these numbers can be substituted in Eq. (8), giving us the dimensionless coefficients, a and b.

Our study presumed the residual stress field known by this method, and obtained the relieved strain by the finite element analysis. We increased the depth of the hole in six steps for the analysis, and obtained the values of a and b, as shown in Fig. 4.

3. Finite Element Analysis of Inclined Hole

3.1 Analysis model and method

In this study, we use the analysis model illustrated in Fig. 5. The hole drilling strain gage was attached to the center of the plate with

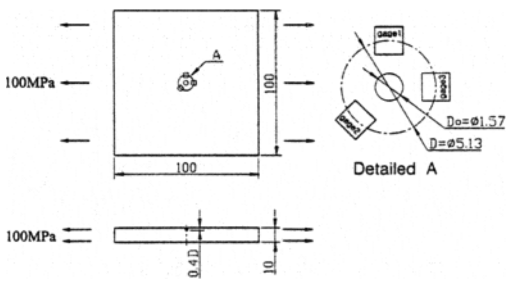


Fig. 5 Model configuration

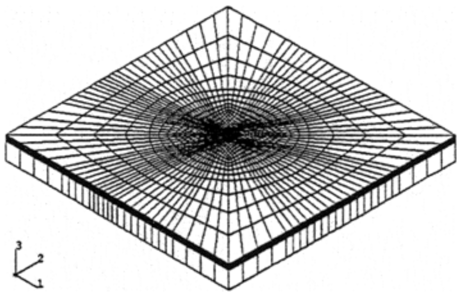
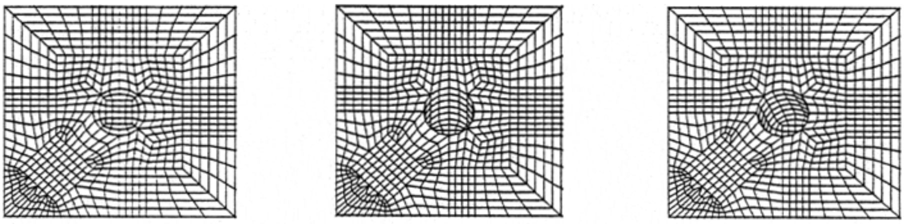


Fig. 6 3-Dimensional finite element model

uniaxial tensile stress. The shape of strain gage was set according to the widely used TEA-XX-062RK-120 of Measurements Group, the hole diameter D_o was set to 1.57 mm, and the diameter of strain gage D to 5.13 mm. In order to know the exact error, we had to calculate the residual stress based on the presumed stress. So we set the stress of σ_x to 100 MPa, σ_y and τ_{xy} to 0 by applying a uniaxial tensile stress of 100 MPa before drilling the hole. The finite element mesh was three-dimensional in order to correspond to the strain gage model, and was divided into 7 layers to allow for the removal of some hole elements while drilling the hole. As to the material properties, the Young's modulus E of 205 GPa and Poisson's ratio ν of 0.3 were used. Fig. 6 shows the finite element mesh used in this analysis, and Fig. 7 shows the details of the strain gage before the drilling, after drilling a vertical hole, and after drilling an inclined hole. Particularly, one strain gage is composed of 48 elements. The average strain of 48 elements was calculated, and it is considered as the strain at each position of strain



(a) Before drilling (b) Vertical drilling (c) Inclined drilling

Fig. 7 Strain gage and hole part

Table 1 Verification results for finite element model

Applied residual stress (MPa)		HDM analysis result			Error in stress (%)
σ_x	σ_y	σ_{max} (MPa)	σ_{min} (MPa)	α (deg)	
100	0	99.95	0.0	0.07	0.05
100	100	99.43	99.43	0.01	0.57
100	−100	100.09	−100.04	0.01	0.09

gage. 14,210 8-nodal solid elements were applied as the elements and the number of nodes was 16, 648. The program used in this finite element study is ABAQUS ver. 5.8.

To analyze the errors due to the inclined hole using this model, the inclined angle was set to 3°, 6°, 9°, and 12°. We also analyzed the errors according to the change of inclined direction by changing the direction of the inclination, 30° interval clockwise from No. 1 strain gage.

3.2 Verification of analysis model

First, we conducted the analysis of residual stress when the hole was not inclined in order to verify the feasibility of the finite element mesh used in the analysis. We calculated the average strain of each element at each strain gage before and after drilling the hole, then put this value into Eq. (11), and came up with the maximum stresses and the direction.

$$\sigma_1,\ \sigma_2=\frac{\varepsilon_3+\varepsilon_1}{4\bar{A}}\pm\frac{\sqrt{(\varepsilon_3-\varepsilon_1)^2+(\varepsilon_3+\varepsilon_1-2\varepsilon_2)^2}}{4\bar{B}}$$

$$\alpha=\frac{1}{2}\tan^{-1}\left(\frac{\varepsilon_3+\varepsilon_1-2\varepsilon_2}{\varepsilon_3-\varepsilon_1}\right) \quad (11)$$

$\bar{A}$ and $\bar{B}$ were calculated by substituting the values of a and b, when z/D was 0.4 in Fig. 4, into Eq. (8). As a result of the analyses for the three different distributions of residual stresses, the error was within 1% when the hole was not inclined and the Table 1 shows such results.

4. Analysis Results and Proposed Correction Equations

4.1 Analysis of errors due to inclined hole

Figure 8 shows the errors due to the changes in the direction of inclination and the inclined angle. Based on the maximum value of the analysis, the maximum error was 10% when the inclined angle of the hole was 12°. As the inclined angle increased, in Fig. 8, the errors included in the measured residual stress also increased. When the hole was inclined in the direction from 0° to 180°, the residual stress was overestimated compared with the actual one, and beyond 180°, the residual stress was underestimated. Especially, when the hole was inclined in the direction of 90° or 270°, the error showed its maximum value. This is because the hole was inclined in the same direction as the direction of maximum residual stress, and the strain at the strain gage became maximum). On the other hand, when the hole was inclined in the direction of 0° or 90°, the error was the minimum because the change in strain was relatively small.

Figure 9 shows the errors for various directions of inclination. The error tends to increase linearly with the inclined angle.

4.2 Proposed of correction equations

When the residual stress is measured with the hole drilling method, the measurement includes a certain level of error when the hole is inclined.

Table 2 Verification result for the correction equations

Applied residual stress (MPa)	Inclined angle (deg)	Direction of inclination (deg)	Analysis result			
			Before correction		After correction	
			Residual stress (MPa)	Error (%)	Residual stress (MPa)	Error (%)
100	10	80	105.9	5.9	100.1	0.1
−200	15	250	−178.2	10.9	−197.6	1.2
300	10	90	322.0	7.3	299.1	0.3

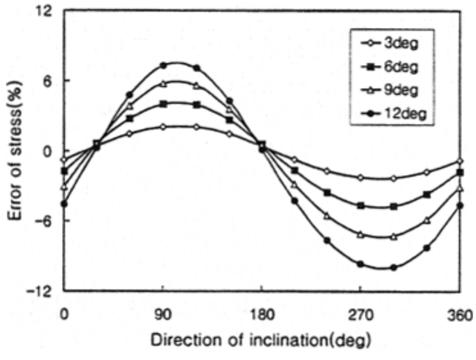


Fig. 8 Error in stress versus the direction of inclination

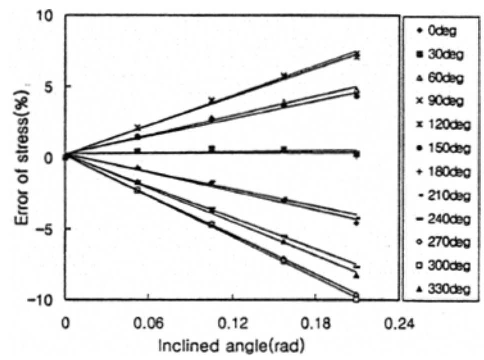


Fig. 9 Error in stress versus inclined angle for various directions of inclination

Since it is impossible to remeasure the residual stress on the same part, it is necessary to correct the errors due to the inclined hole in order to obtain exact residual stress. There is a linear relation between the inclined angle and the error in Fig. 9, so the relation between the inclined angle and the error in the uniaxial residual stress field could be written as in Eq. (12).

$$error(\%) = m \times \psi \tag{12}$$

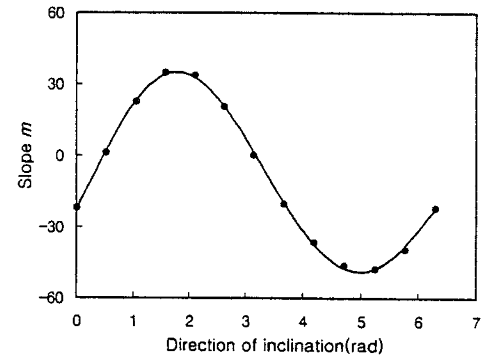


Fig. 10 Slope m of the curves in Fig. 9

where m : slope

ψ : inclined angle(rad)

The slope m in Eq. (12) varies as the direction of inclination changes. Figure 10 shows the slope m according to the direction of inclination, obtamed by a curve fitting of the straight lines in Fig. 9, with the least squares method. The curve in Fig. 10 can be curve fit with a quintic polynomial as in Eq. (13). Here, the coefficient of determination R^2 is 0.988, which demonstrates a reasonable curve fitting.

$$m = -0.2224\theta^5 + 3.6275\theta^4 - 17.381\theta^3 + 14.210\theta^2 + 43.773\theta - 22.412 \tag{13}$$

where θ : inclined direction(rad)

Consequently, when residual stress is measured with an inclined hole, the angle and the direction of the inclined hole should be checked. Then, the slope m could be drawn from Eq. (13), and by substituting the slope into Eq. (12), the value of error can be obtained. Therefore, when the hole is made with an inclination, after correcting the error, the true value of residual stress, Strue, could be found as in Eq. (14).

$$S_{true}=S_{measure}\left(1-\frac{error(\%)}{100}\right) \tag{14}$$

The size of error and the true value of residual stress when the hole is inclined were calculated from Eqs. (12)~(14), and the utility of these equations was examined in several cases. We compared the uncorrected and the corrected results in Table 2. In the hole drilling method, the errors due to the inclined hole were around 10%, but after the correction using Eqs. (12)~(14), the errors were reduced to around 1%. Therefore, the correction using these equations is proved to be effective.

5. Conclusions

In this study, by using the finite element method, we performed an analysis of errors in stress measurement by the hole drilling method due to inclined holes when uniaxial residual stresses exist. Also, we postulated correction equations for errors due to inclined holes. Then, we came up with the following conclusions after studying the validity of the proposed equations.

(1) The error in measurement increases linearly with the inclined angle of the hole.

(2) The error due to inclined holes is the greatest when the direction of inclination of the hole coincides with the direction of the maximum residual stress.

(3) We proposed correction equations which could correct the errors that vary according to the angle and direction of the inclined hole.

(4) The error in the measurement of residual stress could be reduced to around 1% by using the correction equations proposed in this study.

Acknowledgements

The authors are grateful for the support provided by a grant from the Korea Science & Engineering Foundation (KOSEF) and the Safety & Integrity Research Center at the Sungkyunkwan University.

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